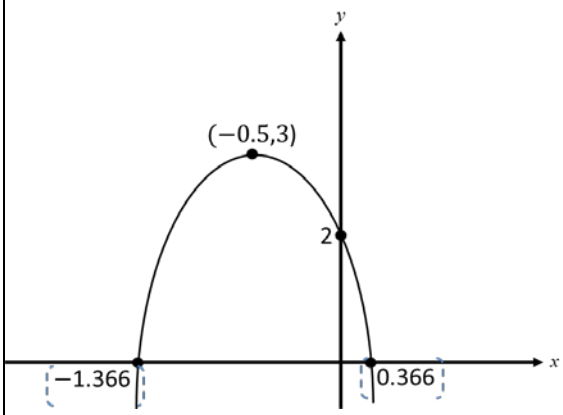
Additional Mathematics Paper 2

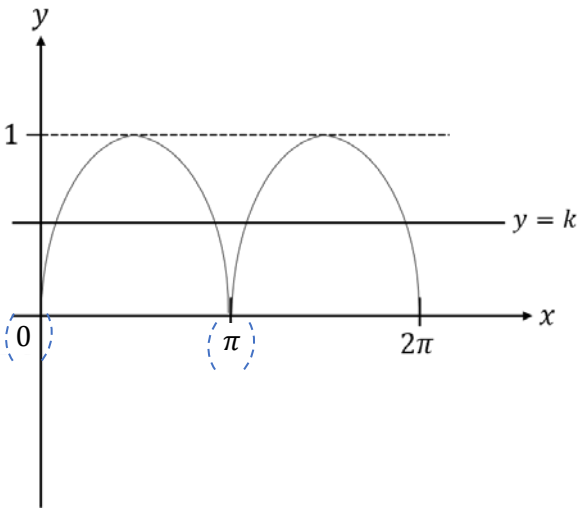
SPMRSM 2021

ANSWER SCHEME

No	Solution	Scheme	Sub marks	Marks
1 (a)	$f(x) = -4\left[x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - \frac{2}{4}\right]$ OR $f(x) = -4\left(x + \frac{-4}{2(-4)}\right)^2 + 2 - \frac{(-4)^2}{4(-4)}$ or $f(x) = -4\left(x + \frac{-4}{2(-4)}\right)^2 + \frac{4(-4)(2) - (-4)^2}{4(-4)}$ $-4\left(x + \frac{1}{2}\right)^2 + 3$	K1 Use completing the square method N1 $-4\left(x + \frac{1}{2}\right)^2 + 3$	2	
(b)		P1 Shape P1 Maximum point (-0.5,3) and y-intercept	2	
(c)	$-4\left(x + \frac{1}{2}\right)^2 + 3 = 0.44$ **or** $-4x^2 - 4x + 2 = 0.44$ 0.8	K1 Equate $f(x)$ to 0.44 N1	2	6

2	$4x+2y+2z=68$ or equivalent $y-x=4$ or equivalent $\frac{z}{x}=\frac{3}{4}$ or equivalent $2x+y+z=34$ $(-)\ y-x=4$ $3x+z=30$ OR any other valid method $3x+z=30$ $(-)\ 4z-3x=0$ $5z=30$ $z=6$ $x=8$ $y=12$	P1 Produce at least two correct equations K1 Eliminate one variable K1 Eliminate two variables N1 First value obtained N1 Second and third value obtained		5
3 (a)	0.7688//0.7689	N1	1	
(b)	$16^{*}(0.7688)$ $Perimeter = 16^{*}(0.7688)+12$ $=24.3$	K1 Use $s=r\theta$ K1 $^{*}s+12$ N1	3	
(c)	1.424 $Area=\frac{1}{2}\times 16^{2}\times^{*}1.424$ $=182.23 \leftrightarrow 182.27$	P1 $\frac{3}{5}\times(\pi-^{*}0.7688)$ K1 Use $A=\frac{1}{2}r^{2}\theta$ N1	3	7

4 (a)	$\frac{\cos^2 x}{\sin x} + \sin x$ $= \frac{\cos^2 x + \sin^2 x}{\sin x} \times \frac{2\cos x}{2\cos x}$ $= \frac{2\cos x}{2\sin x \cos\ x}$ $= 2\cos x\left(\frac{1}{\sin 2x}\right)$ $= 2\cos x \cos ec\ 2x$ **ALTERNATIVE** $2\cos x \cos ec\ 2x$ $= 2\cos x\left(\frac{1}{\sin 2x}\right)$ $= \frac{2\cos x}{2\sin x \cos\ x}$ $= \frac{1}{\sin x}$ $= \frac{\cos^2 x + \sin^2 x}{\sin x}$ $= \frac{\cos^2 x}{\sin x} + \sin x$	K1 multiply $\frac{2\cos x}{2\cos x}$ K1 Use $\sin 2x = 2\sin x\cos x$ **and** $\cos^2 x + \sin^2 x = 1$ N1 K1 Use $\cos ec2x = \frac{1}{\sin 2x}$ **and** $\sin 2x = 2\sin x\cos x$ K1 Use $1 = \cos^2 x - \sin^2 x$ N1	3 3	

(b)	(i)  Note: ignore graph outside the range	P1 Shape of $sin\ x$ P1 Shape of absolute $sin\ x$ P1 Amplitude 1	3	
	(ii) Refer Q 4 (b) (i) $0 < k < 1$	P1 Accept horizontal line within range $0 < y < 1$ N1	2	8

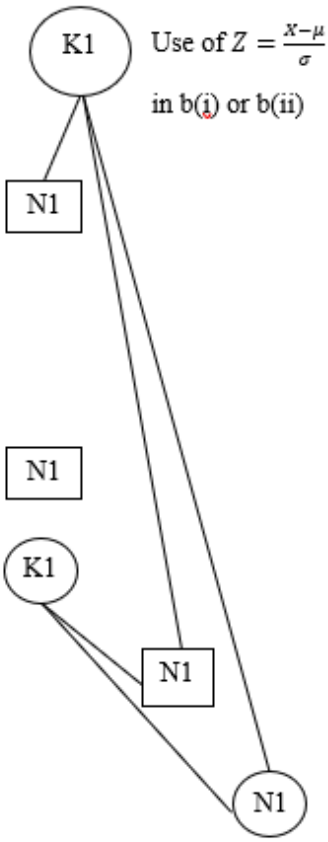
5 (a)	i) $\overrightarrow{AC} = 4\underset{\sim}{x} + \frac{1}{2}\underset{\sim}{y}$ ii) $\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$ $= (4\underset{\sim}{x} + \frac{1}{2}\underset{\sim}{y}) + \frac{3}{2}(\frac{1}{2}\underset{\sim}{y})$ $= 4\underset{\sim}{x} + \frac{5}{4}\underset{\sim}{y}$ or $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$ $= 4\underset{\sim}{x} + \frac{5}{2}(\frac{1}{2}\underset{\sim}{y})$ $= 4\underset{\sim}{x} + \frac{5}{4}\underset{\sim}{y}$	Write triangle law $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ **or** $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$ **or** $\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$ **or** $\overrightarrow{AF} = \lambda\overrightarrow{AC} + \overrightarrow{GF}$ **or** $\overrightarrow{AF} = \mu\overrightarrow{AD} + \overrightarrow{EF}$ K1 N1 $4\underset{\sim}{x} + \frac{1}{2}\underset{\sim}{y}$ N1 $4\underset{\sim}{x} + \frac{5}{4}\underset{\sim}{y}$	3	
(b)	$\overrightarrow{AG} = \lambda\overrightarrow{AC}$ $\overrightarrow{AF} = \lambda\overrightarrow{AC} + \overrightarrow{GF}$ $= \lambda^{*}\left(4\underset{\sim}{x} + \frac{1}{2}\underset{\sim}{y}\right) + \frac{3}{2}\underset{\sim}{y}$ $= 4\lambda\underset{\sim}{x} + (\frac{1}{2}\lambda + \frac{3}{2})\underset{\sim}{y}$ $\overrightarrow{AF} = \mu\overrightarrow{AD} + \overrightarrow{EF}$ $= \mu^{*}\left(4\underset{\sim}{x} + \frac{5}{4}\underset{\sim}{y}\right) + 3\underset{\sim}{x}$ $= (4\mu + 3)\underset{\sim}{x} + (\frac{5}{4}\mu)y$ $4\lambda\underset{\sim}{x} + (\frac{1}{2}\lambda + \frac{3}{2})\underset{\sim}{y} = (4\mu + 3)\underset{\sim}{x} + (\frac{5}{4}\mu)\underset{\sim}{y}$ $4\lambda = 4\mu + 3$ $\frac{1}{2}\lambda + \frac{3}{2} = \frac{5}{4}\mu$ @ $2\lambda + 6 = 5\mu$ $\lambda = \frac{13}{4}$ or $\mu = \frac{5}{2}$ $\lambda = \frac{13}{4}$ and $\mu = \frac{5}{2}$	N1 N1 K1 Equate coefficient of $\underset{\sim}{x}$ and $\underset{\sim}{y}$, and solve simultaneous eq. N1 N1	5	8

6 (a)	$\frac{du}{dx}=3$ **or** $\frac{dv}{dx}=4(x^2+5)^3(2x)$ $\frac{dy}{dx}=3x(4)(2x)(x^2+5)^3+3(x^2+5)^4$ $\frac{dy}{dx}=3(x^2+5)^3(9x^2+5)$ 9	K1 Diff 3x or $(x^2+5)^4$ wrt x K1 use product rule N1	3	
(b)	$r=\frac{1}{3}h$ **or** $\delta V=-0.5\pi$ $V=\frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$ $V=\frac{1}{27}\pi h^3$ $\frac{dV}{dh}=\frac{1}{9}\pi h^2$ $\frac{1}{27}\pi h^3=8\pi$ $h=6$ $*-0.5\pi=\frac{*1}{9}\pi(*6)^2\times\delta h$ $\delta h=-0.125$ // $-\frac{1}{8}$ Note: If $\frac{dv}{dt}=\frac{dv}{dh}\times\frac{dh}{dt}$ Award K0 N0	P1 K1 Derive V in term of h K1 Differentiate $^*V$ wrt h K1 Use $\frac{*\delta V}{\delta h}\approx\frac{*dV}{dh}$ to find δh N1	5	8

7 (a)	$\frac{a}{1-r} = 6$ or $a + ar = \frac{16}{3}$ $6(1-r) = \frac{16}{3(1+r)}$ $r = \frac{1}{3}$ *and* $r = -\frac{1}{3}$	P1 $\frac{a}{1-r} = 6$ or $a + ar = \frac{16}{3}$ seen K1 Solve simultaneously N1	3	
(b) (i)	$S_{31} = 30\left(\frac{1.15^{31}-1}{1.15-1}\right)$ $S_{31} = 15028 / / 15029$	K1 Use formula S_n for geometric progression with correct *a* and *r* N1 15028 // 15029		
(ii)	$2284(r)^{30} = 45464$ ~~30 log *r* =log 22.892~~ *r* = 1.105 Yes	K1 Use formula T_n to find *r* with a = 2284 and valid method to find r N1 1.105 N1	5	8

7 (b)	(i) $S_{31} = 30\left(\frac{1.15^{31}-1}{1.15-1}\right)$ $S_{31} = 15028 // 15029$ (ii) $2284(r)^{30} = 45464$ $30 \log r = \log 19.9054$ $r = 1.105$ Yes	K1 Use formula S_n for geometric progression with correct a and r N1 15028 //15029 K1 Use formula T_n to find r with a = 2284 and valid method to find r N1 1.105 N1	5	8

8 (a)	$P(X=1)$ or $[P(Y=n)]$	P1	Seen or implied	
	${}^nC_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^{n-1}$ or ${}^nC_n\left(\frac{1}{3}\right)^n\left(\frac{2}{3}\right)^0$	K1	Use ${}^nC_r p^r q^{n-r}$	
	${}^nC_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^{n-1} = 10\left[{}^nC_n\left(\frac{1}{3}\right)^n\left(\frac{2}{3}\right)^0\right]$	K1		
	$n=5$	N1		4
(b) (i)	$P\left(Z > \frac{40-45}{15}\right)$	K1	Use of $Z=\frac{X-\mu}{\sigma}$	
	$= P(Z > -0.333)$			
	$= 0.6304$ // 0.6306	N1		2
(ii)	$P(X \le t) = 0.1$			
	$P\left(Z \le \frac{t-45}{15}\right) = 0.1$	P1	$Z = (-)1.281$ / / $(-)1.282$	
	$\frac{t-45}{15} = -1.281$// -1.282	K1	Use of Z and equate to $\frac{t-45}{15}$	
	$t = 25.77$ // 25.78//25.79	N1	25.77 // 25.78//25.79	
	Yes	N1		4
				10

8 (b)	**ALTERNATIVE METHOD** (i) $P\left(Z > \frac{40-45}{15}\right)$ $= P(Z > -0.333)$ $= 0.6304$ // 0.6306 $P(X \leq 25)$ $P(Z \leq -1.333) = 0.0913$ 0.0913×100 9.13 // $9 < 10$ Yes		4	

(b)	**ALTERNATIVE METHOD** (i) $P\left(Z > \frac{40-45}{15}\right)$ $= P(Z > -0.333)$ $= 0.6304$ // 0.6306 $P(X \leq 25)$ $P(Z \leq 1.333) = 0.0913$ 0.0913×100 9.13 // $9 < 10$ Yes	K1 N1 N1 K1 N1 N1 Use of $Z = \frac{X-\mu}{\sigma}$ in b(i) or b(ii)	4	

9 (a)

T	- 5	5	10	15	20	25
$\log_{10}P$	0.31	0.55	0.75	0.79	0.90	1.03

N1 Implied
Note : at least two d.p

Plot $\log_{10}P$ against T

K1 Plot $\log_{10}P$ against T with correct axes and uniform scales

*6 points plotted correctly

N1

Draw line of best fit
(All *points correctly plotted)

N1 4

(b) $\log_{10}P = 0.67$

$P = 4.62 \leftrightarrow 4.73$

N1 1

4.62 ↔ 4.73

(c) $\log_{10}P = \left(\frac{1}{2}\log_{10}r\right)T + \frac{1}{2}\log_{10}q$

P1

$log_{10}P = \left(\frac{1}{2}\log_{10}r\right)T + \frac{1}{2}\log_{10}q$

seen or implied

(i) $\frac{1}{2}\log_{10}q = 0.43$

$q = 7.079 \leftrightarrow 7.413$

K1 Use * c $= \frac{1}{2}\log_{10}q$

N1

7.079 ↔ 7.413

(ii) $\frac{1}{2}\log_{10}r = 0.024$

$r = 1.116 \leftrightarrow 1.118$

K1 Use * m $= \frac{1}{2}\log_{10}r$

N1 5 10

10 (a)	$-\frac{3}{2}m_2 = -1$ $m_2 = \frac{2}{3}$ $y-3 = {}^{*}\frac{2}{3}(x-(-5))$ $y = \frac{2}{3}x + \frac{19}{3}$ *or equivalent*	K1 K1 N1	Use $m_1 m_2 = -1$ ${}^{*}-\frac{3}{2}m_2 = -1$ Use m_2 and coordinate M(-5,3) to form equation $y-3 = {}^{*}\frac{2}{3}(x-(-5))$	3
(b)	${}^{*}y = \frac{2}{3}(-8) + {}^{*}\frac{19}{3}$ $k = 1$ $(-5,3) = \left(\frac{x+(-9)}{2}, \frac{y+9}{2}\right)$ $C(-1,-3)$ $\frac{1}{2}\left\|\left[(-9)({}^{*}1)+(-8)({}^{*}-3)+({}^{*}-1)(9)+(1)(9)\right]-\left[(9)(-8)+({}^{*}1)(-1)+({}^{*}-3)(1)+(9)(-9)\right]\right\|$ $= 86$	K1 K1 K1 N1	Substitute B(-8,k) into ${}^{*}y = \frac{2}{3}(-8) + {}^{*}\frac{19}{3}$ $k = 1$ Use midpoint formula $(-5,3) = \left(\frac{x+(-9)}{2}, \frac{y+9}{2}\right)$ $C(-1,-3)$ Note: Accept correct answer without working Use area formula 86	4
(c)	$PA = 10$ $\sqrt{(x+9)^2+(y-9)^2} = 10$ $x^2+y^2+18x-18y+62 = 0$	K1 K1 N1	Seen or implied Use distance formula	3
				10

11 (a)	$2x = 6$ $x = 3$ Area of triangle, $A_1 = \frac{1}{2}(12)(2)$ $= 12$ $A_2 = \int_0^{*3} (x^2 + 3)dx$ $= \left[\frac{x^3}{3} + 3x\right]_0^{*3}$ $= \left[\left(\frac{*3}{3} + 3(*3)\right) - 0\right]$ $*18 - *12$ 6	K1 Differentiate and equate to 6 N1 K1 Find area of triangle K1 Integrate $x^2 + 3$ w.r.t x K1 Use limit 0, 3 K1 *A2 – *A1 (A2 > A1) N1	7	
(b)	$V = \pi\int_{*3}^{12}(y-3)dy$ $= \pi\left[\frac{y^2}{2} - 3y\right]_{*3}^{12}$ $= \pi\left[\left(\frac{12^2}{2} - 3(12)\right) - \left(\frac{*3^2}{2} - 3(*3)\right)\right]$ $= 40.5\pi$ // $40\frac{1}{2}\pi$ // $\frac{81}{2}\pi$	K1 Integrate $\pi\int(y-3)\,dy$ K1 Use limit *3, 12 N1	3	10

12 (a)	$\frac{Q_{19}}{5550} \times 100 = 105$ $5827 \,//\, 5827.50 \,//\, 5828$	K1 Use $\frac{Q_1}{Q_0} \times 100$ N1	2	
(b) (i)	$\frac{146.4 \times 100}{120}$ 122	K1 N1 122		
	(ii). Increase 22%	N1 Increase 22%	3	
(c) (i)	$\frac{6710}{6100} \times 100$ $r = 110$	K1 Use $\frac{Q_1}{Q_0} \times 100$ N1		
	(ii) $\frac{(\,^{*}110 \times q) + (105 \times 2) + (\,^{*}122 \times 8) + (116 \times 7)}{q + 2 + 8 + 7} = 116.4$ $q = 3$	K1 Use $\frac{\sum Iw}{\sum w}$, accept in terms of r and q K1 Equate $\frac{\sum Iw}{\sum w}$ *to* 116.4 *and solve* N1 $q = 3$	5	10

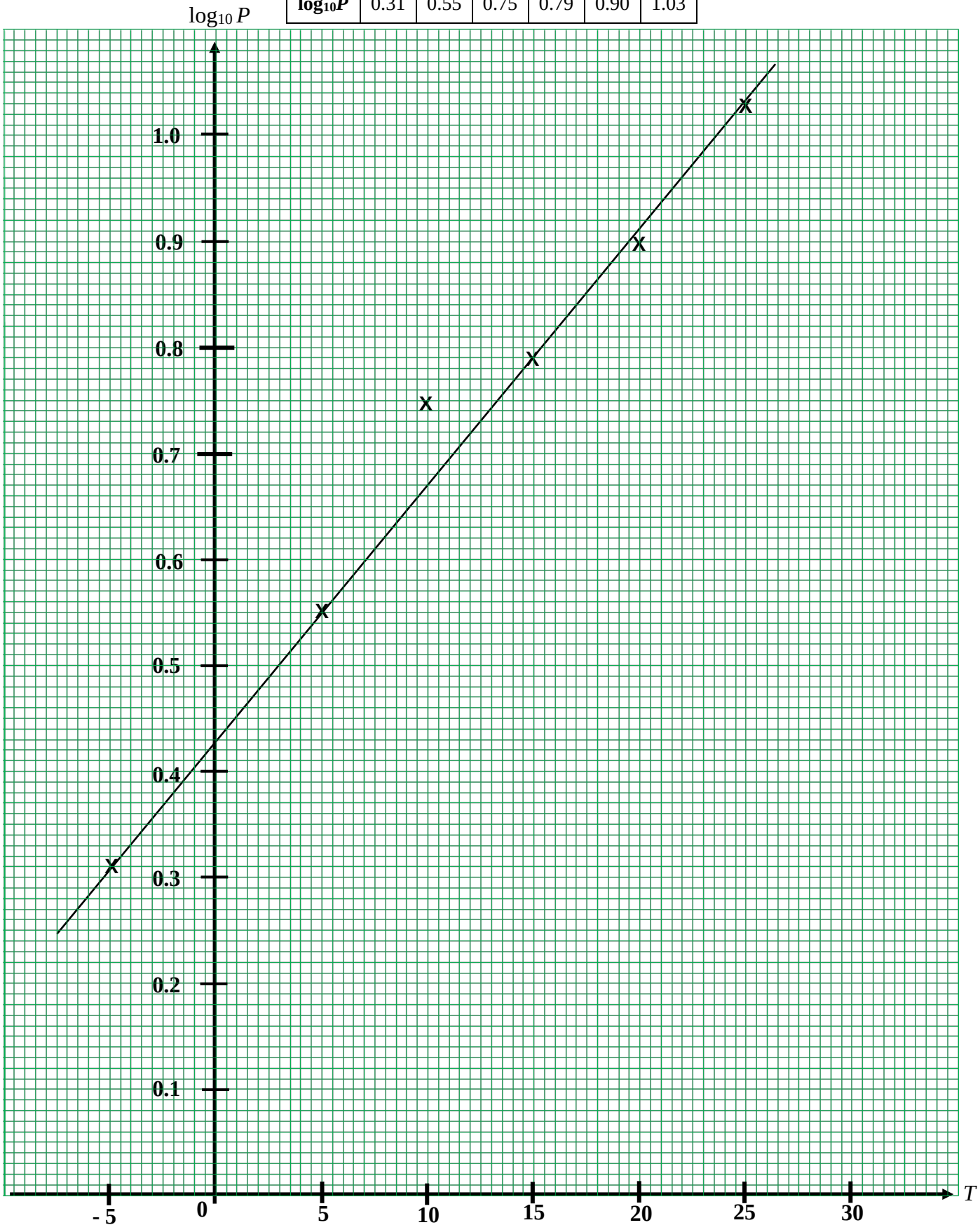
13 (a) (i)	$\frac{\sin \angle PRS}{13.58} = \frac{\sin 26.6}{8.599}$ $\angle PRS = 135$	K1 N1	Using sine rule to find $\angle PRS$ or PM	
(ii)	$\angle PSM = 180 - 26.6 - 45$ $\frac{PM}{\sin {}^{*}108.4} = \frac{13.58}{\sin {}^{*}45}$ $PM = 18.22\ cm$	K1 N1	Find $\angle PSM$ $PM = 18.22$	4
(b)	$\frac{PR}{\sin {}^{*}18.4} = \frac{8.599}{\sin 26.6}$ 12.21^2 $= {}^{*}6.062^2 + \ 7.813^2$ $- 2({}^{*}6.062)(7.813)\cos \angle PRQ$ $\angle PRQ = 122.79$	K1 K1 N1	Finding PR with the valid method Use cosine rule	3
(c)	Area $\triangle PQR$ $= \frac{1}{2}({}^{*}6.062)\ \ (7.813)(sin {}^{*}122.79)$ **or** Area$\triangle PMS$ $= \frac{1}{2}(\ 13.58)({}^{*}18.22)(sin\ \ 26.6)$ Area pentagon $PQRMS$ $\frac{1}{2}({}^{*}6.062)\ \ (7.813)(sin {}^{*}122.79) +$ $\frac{1}{2}(\ 13.58)({}^{*}18.22)(sin\ \ 26.6)$ 75.3 // 75.31	K1 K1 N1	Use $\frac{1}{2}ab\ \sin c$ Sum of ${}^{*}\triangle PQR$ and ${}^{*}\triangle PMS$	3
				10

14 (a)	$s = -\frac{4t^3}{3} + 28t^2 - 160t$ $a = -8t + 56$	N1 N1	2	
(b)	$*(-8t + 56) = 0$ $t = 7$ $*[-4(*7)^2 + 56(*7) - 160]$ 36 No	K1 Use $a = 0$ and solve Substitute t = 7 into v K1 N1 N1	4	
(c)	$-4t^2 + 56t - 160 = 0$ $t = 4 \ \ and \ t = 10$ $s(*4) = *\left[-\frac{4(*4)^3}{3} + 28(*4)^2 - 160(*4)\right]$ $s(*10) = *\left[-\frac{4(*10)^3}{3} + 28(*10)^2 - 160(*10)\right]$ Total distance travelled $\left\lvert *\left[-\frac{4(*4)^3}{3} + 28(*4)^2 - 160(*4)\right]\right\rvert$ -* $\left\lvert\left[-\frac{4(*10)^3}{3} + 28(*10)^2 - 160(*10)\right]\right\rvert$ 144	K1 Use v=0 to find t K1 Find s(*4) or s(*10) K1 $\lvert S_4\rvert - \lvert S_{10}\rvert$ N1 144	4	10

15 (a)	$20x + 30y \geq 1800$ or equivalent $y \leq 2x$ or equivalent $x \leq 45$ or equivalent	N1 N1 N1	3	
(b)	Graph (Attachment) Draw correctly at least one straight line from *inequalities *x* and/or *y* Draw correctly at the *straight line from *inequalities *x* and/or *y* Note: accept dotted line Correct shaded region	K1 N1 N1	3	
(c)	(i) $40 \leq y \leq 60$ (ii) (22.5,45) 15(*22.5) + 20(*45) 1237.50 Note: SS-1 once if in b(i) does not use given scale (ii) axis interchange	N1 N1 K1 Substitute any point in the *shaded region into 15x + 20y N1	4	10

Graph for Question 9

T	- 5	5	10	15	20	25
$\log_{10} P$	0.31	0.55	0.75	0.79	0.90	1.03



Graph for Question 15